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# Applications in Trigonometry

## Trigonometric Ratios of Special Angles

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

Note that and , i.e., , and . Also note that , i.e., etc. You may remember the trigonometric ratios in the sequence presented, or derive them based on the angles you know with these handy formulae.

### Worded Example 1A picture containing diagram Description automatically generated

In , and . is a point on such that . If , find the length of . Leave your answer in surd form and no calculator is allowed.

Solution

*(given)*

## Gradients

Let a straight line represent the inclined plane, and the vertical distance and the horizonal distance.

Gradients are usually represented in the form of , i.e., . This can be done by simplifying the ratio.

The inclination of a plane is the angle between the inclined plane and the horizontal plane. The gradient can be found given the inclination and vice versa.

Note that on a contour map, the vertical distance between two points is given by counting the contour lines and the horizontal distance is given by measuring the length with conversion according to the scale given.

### Worked Example 2

Chart

Description automatically generatedIn the figure, the gradient of plane *AD* is 1 : 8 and the inclination of plane *DE* is . If C is the midpoint of *AC,* Find the gradient of plane *AE.*

Solution

Let *AC = CB = x.*

Height of plane *AD* =

Height of plane *DE* =

The gradient of plane *AE* = = =

## Angles of Elevation and Depression

When one looks above the horizontal, the angle between the line of sight and the horizontal is the *angle of elevation.* When one looks below the horizontal, it is the *angle of depression.*

### Exercise 1

Carol and Emily are walking toward a building in opposite directions. They look at the top of the building with the same angle of elevation . Let *x* be the height of Carol, *y* be the distance between Carol and the building and *z* the distance between Emily and Carol. If *y =* and , prove that the ratio between the height of Emily and Carol is *1 : 2.* This is left as an exercise to the reader.

## Bearings

There are four main bearings: east (E), south (S), west (W), north (N); there are other four between them: north-east (NE), south-east (SE), south-west (SW).

Chart, radar chart

Description automatically generated

### Compass Bearing

A compass bearing is an expression of the direction in either one of the following form where :

or or or

The compass bearing can always be found by the difference of angles.

### True Bearing

A true bearing is written as an angle measured from the north to the point in a clockwise direction where . Note that the integral part of must be written 3 DIGITS! , i.e., has to be ! A true bearing which is an obtuse angle can always be given by the sum of some angles.

### Exercise 2

Ship *A* leaves port *P* at 9:30 a.m. and ship *B* leaves port *Q* at 10:00 a.m. on the same day. *A* sails at a constant speed of 24 km/h in the direction of and expects to arrive at island *R* at 2:00 p.m. on the same day. *B* sails at a constant speed in the direction of and expects arrive at island *S* at 4:00 p.m. the same day. The bearing of *Q* from *P* is . The bearing of *S* from *R* is . The distance between *P* and *Q* is 71 km. The distance between Q and S is 163 km.

Suppose *B* strikes a reef and stops moving at 1:00 p.m. The captain of ship *B* immediately sends out an SOS. An 1:30 p.m., ship *A* starts to sail a constant speed of 40 km/h to ship *B* is a straight line.

(a.) Find the reduced bearing of ship *B* from ship *A* at 1:30 p.m.

(b.) Can ship *A* arrive at the location of ship *B* before 3:00 p.m.? Explain your answer.

Reminder: Do not misinterpret speed as distance!

# Probability

## Theoretical Probability

A sample space *S* is the set of all possible outcomes. The probability of a sample space is defined to equal to 1, i.e., , that is, the sum of probability of every outcome is 1. An event *A* is the set of all favorable outcomes. The theoretical probability of an event is:

For example, when you toss a fair coin, the probability of getting a head is as there is only one favorable outcomes, i.e., getting a head, while there is other possible outcome, i.e., getting a tail.

Note that, for example, when you choose a number from 1 to 10 inclusively, the probability for the number to be divisible by 2 or 3 is instead of . Although there are 5 numbers that are divisible by 2 (2, 4, 6, 8, 10) and 3 numbers divisible by 3 (3, 6, 9), in the number *6* is counted *twice*, but it is only *one* favorable outcome. Note that for *non-disjoint events*, events which share the same outcome(s) like the case before, be careful when counting the number of favorable outcomes! Also note that the probability for an impossible event is 0.

### Tree Diagrams

A *tree diagrams* can be drawn to analyze outcomes that come in order. For example, if you want to investigate the probability of getting at least two heads from tossing a coin three times, the order of each outcome matters.

Chart, diagram

Description automatically generated

Let each node in the tree be either a 'H' (head) or a 'T' (tail). Under each node, there are the two nodes 'H' and 'T' until the depth of the tree is 3, i.e., the coin is tossed three times. A tree diagram can be read from the left to the right by following each path down. For example, the 'H', 'H', 'H' path represents the 'HHH' result. There are four favorable outcomes (HHH, HHT, HTH, THH) and 8 possible outcomes in total. Therefore the probability required is .

### Table

A *table* works similarly as a tree diagram. It can be used when there are only two successive outcomes.

### Worked Example 3

There are two bags *A* and *B. A* contains 2 green balls, 1 yellow ball, and 1 black ball. *B* contains 1 green ball, 2 yellow balls and 1 black ball. A ball is drawn at random from each bag. Find the probability that the two balls drawn are of different colors.

Answer

We first list all the possible outcomes in a table. (We use G to denote a green ball, Y to denote a yellow ball and B to denote a black ball.)

|  |  |  |  |
| --- | --- | --- | --- |
| Bag A | | | |
| G | G | Y | B |
| Bag B | G | GG | GG | GY | GB |
| Y | YG | YG | YY | YB |
| Y | YG | YG | YY | YB |
| B | BG | BG | BY | BB |

## Geometry Problems

Note that for geometry problems, the likelihood of an event is determined by the proportion.

### Worked Example 4

Suppose there is a dartboard which is made up of two concentric circles of radii 9 cm and 12 cm respectively. If a dart is thrown at random and hits the board, find the probability that the dart hits the outer ring of the dartboard.

Solution

Area of the dartboard = .

Area of the outer ring = .

The probability that the dart hits the outer ring = .

### Exercise 3

Chart, pie chart

Description automatically generatedAmy spins the lucky wheel as shown in the figure and

gets the gift where the pointer stops. Find the probability that

Amy:

(a.) wins a computer.

(b.) wins a keyboard.

## Experimental Probability

For experimental probability, the number of favorable outcomes is given by counting the experimental results and the number of all possible outcomes is given by the total test cases.

### Exercise 4

A dice is thrown 200 times. The results are recorded in the following table:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Number | 1 | 2 | 3 | 4 | 5 | 6 |
| Outcomes | 18 | 56 | 4x | 35 | 12 | 2x + 1 |

(a.) Find the value of x.

(b.) Find the experimental probability of getting a number less than or equal to 3.

## Expected Value

In probability, the expected value *E[X]* is a generalization of the weighted average:

where *p* is the probability and *x* is the random variable (value). For example, in the upper case, the expected value *E[X]* is:

### Exercise 5

Referring to exercise 3, if the angle of the sector also represents the price of the gift. Find the expected value of the price of the gift.